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ABSTRACT

The Comprehensive School Mathematics Program (CSMP) is a program of CEMREL, Inc., one of the national educational laboratories, and was funded by the National Institute of Education (NIE). Its major purpose is the development of curriculum materials for the elementary grades. Beginning in 1973, CSMP ran an extended pilot trial of its elementary program. The 1976-77 period was the fourth year. This material describes the results of some individually conducted tests with third graders in the St. Louis region. The purpose of this testing was to achieve a deeper understanding of the mathematical processes used by CSMP versus non-CSMP students than paper-and-pencil testing allows. It is noted that a great deal of information has been summarized very briefly. Results indicated very real differences in favor of CSMP students on a relatively small number of items, but for many of the tasks the differences were not considered large enough to be of importance, particularly in view of the small numbers of pupils tested. It is noted that the study was exploratory, with hopes that a more efficient procedure would be adopted in the future to further probe some of the findings. (MP)



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# EXTENDED PILOT TRIALS OF THE COMPREHENSIVE SCHOOL MATHEMATICS PROGRAM:

## EVALUATION REPORT SERIES

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# Evaluation Report 4–B–3
# Individually Administered Problems
# Third Grade

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SE040184

# Extended Pilot Trial of the Comprehensive School Mathematics Program

Evaluation Report 4-B-3

Individually Administered Problems

Third Grade

Martin Herbert

November, 1977

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## Description of Evaluation Report Series

The Comprehensive School Mathematics Program (CSMP) is a program of CEMREL, Inc., one of the national educational laboratories, and is funded by the National Institute of Education. Its major purpose is the development of curriculum materials for grades K-6.

Beginning in September, 1973, CSMP began an extended pilot trial of its Elementary Program. The pilot trial is longitudinal in nature; students who began using CSMP materials in kindergarten or first grade in 1973-74, were able to use them in first and second grades respectively in 1974-75, and so on in subsequent years. Hence the adjective "extended".

The evaluation of the program in this extended pilot trial is intended to be reasonably comprehensive and to supply information desired by a wide variety of audiences. For that reason the reports in this series are reasonably non-technical and do not attempt to widely explore some of the related research issues. The list of reports from the first two years of the extended pilot trial is given on the next page. The most comprehensive of these are the following:

1-A-1: Overview, Design and Instrumentation
1-A-3: Final Summary Report, Year 1
2-A-1: Final Summary Report, Year 2
3-B-1: Summary of Second and Third Grade Test Data Year 3
4-A-1: Final Summary Report, Year 4
and 4-A-2: Summary of Grades K-3 Evaluation

## Evaluation Report Series

| | |
|---|---|
| Evaluation Report 1-A-1 | Overview, Design and Instrumentation |
| Evaluation Report 1-A-2 | External Review of CSMP Materials |
| Evaluation Report 1-A-3 | Final Summary Report Year 1 |
| Evaluation Report 1-B-1 | Mid-Year Test Data: CSMP First Grade Content |
| Evaluation Report 1-B-2 | End-of-Year Test Data: CSMP First Grade Content |
| Evaluation Report 1-B-3 | End-of-Year Test Data: Standard First Grade Content |
| Evaluation Report 1-B-4 | End-of Year Test Data: CSMP Kindergarten Content |
| Evaluation Report 1-B-5 | Test Data on Some General Cognitive Skills |
| Evaluation Report 1-B-6 | Summary Test Data: Detroit Schools |
| Evaluation Report 1-C-1 | Teacher Training Report |
| Evaluation Report 1-C-2 | Observations of CSMP First Grade Classes |
| Evaluation Report 1-C-3 | Mid-Year Data from Teacher Questionnaires |
| Evaluation Report 1-C-4 | End-of-Year Data from Teacher Questionnaires |
| Evaluation Report 1-C-5 | Interviews with CSMP Kindergarten Teachers |
| Evaluation Report 1-C-6 | Analysis of Teacher Logs |
| Evaluation Report 2-A-1 | Final Summary Report Year 2 |
| Evaluation Report 2-B-1 | Second Grade Test Data |
| Evaluation Report 2-B-2 | Readministration of First Grade Test Items |
| Evaluation Report 2-B-3 | Student Interviews |
| Evaluation Report 2-C-1 | Teacher Questionnaire Data |
| Evaluation Report 2-C-2 | Teacher Interviews, Second Grade |
| Evaluation Report 2-C-3 | Teacher Interviews, First Grade |
| Evaluation Report 3-B-1 | Second and Third Grade Test Data Year 3 |
| Evaluation Report 3-C-1 | Teacher Questionnaire Data Year 3 |
| Evaluation Report 4-A-1 | Final Summary Report Year 4 |
| Evaluation Report 4-A-2 | Summary of Grades K-3 Evaluation |
| Evaluation Report 4-B-1 | Standardized Test Data, Third Grade |
| Evaluation Report 4-B-2 | Mathematics Applied to Novel Situations (MANS) Test Data |
| Evaluation Report 4-B-3 | Individually Administered Problems, Third Grade |
| Evaluation Report 4-C-1 | Teacher Questionnaire Data, Third Grade |

### Key to Indexing

1-C-2 Observations of CSMP First Grade Classes

"2" refers simply to the number within a given year and type of data

"C" refers to the type of data being reported
- A: Overview, summary and theoretical reports
- B: Student outcomes
- C: Non-test data

"1" refers to the year of the Pilot Study according to the following:

| | Kindergarten | First Grade | Second Grade | Third Grade | . . . |
|---|---|---|---|---|---|
| Year 1 (1973-74) | ▨ | ▨ | | | |
| Year 2 (1974-75) | | ▨ | ▨ | | |
| Year 3 (1975-76) | | | ▨ | ▨ | |

Table of Contents

Introduction . . . . . 1

Setting . . . . . 2

Section A: Numerical Processes . . . . . 3

Section B: Classification and Logical Reasoning . . . . . 12

Summary . . . . . 17

Appendix A: Protocol for Administering Section A . . . . . 21

Appendix B: Protocol for Administering Section B . . . . . 35

## Introduction

In the fall of 1973, the Comprehensive School Mathematics Program (CSMP) began a longitudinal pilot study of its Elementary School Program. Over 100 teachers began using the program, either in first grade or kindergarten. During the 1974-75 and 1975-76 school years (the second and third years of the pilot study) some of these classes continued with CSMP materials in the next grade levels, and many new classes began using CSMP materials. The 1976-77 school year constituted the fourth year of this pilot study, with some of the original classes continuing with CSMP in third and fourth grades. Once again, many new classes began using CSMP at grade levels K through 4.

This report describes the results of some individually conducted tests with third graders in the St. Louis region. The purpose of this task was to get a deeper understanding of the mathematical processes used by CSMP versus Non-CSMP students than paper and pencil allow. A great deal of information - the process of development, the complexity of the eventual format and the mass of student responses - has been summarized very briefly in this report. This study is exploratory and will continue, in one form or another, in higher grades.

## Setting

Two series of individually-administered tests, Section A and Section B, were developed for use with third grade students. They were designed to be used with pairs of students; the students usually alternated in answering questions and occasionally cooperated (or competed). It was thought they were more likely to discuss a certain concept with one another than would a single student with the interviewer.

In the local St. Louis area, where these interviews were carried out, this was part of a larger testing program including the use of standardized mathematics tests and problem solving tests. The experimental design for these other studies involved a total of 12 third grade CSMP classes in three school districts, and 12 Non-CSMP comparison classes. These 12 CSMP classes came from 7 different schools and the 12 Non-CSMP classes came from 6 different schools. Most often there were 2 classes per school. For the purposes of this study, schools and classes were paired so that within pairs, classes were of about equal ability and schools were located in similar and nearby neighborhoods. The number of pairs of students selected from various schools and classes is shown below. It can be seen that all but one CSMP and Non-CSMP school were included in the study.

| | CSMP Classes | Number of Pairs of Students | Non-CSMP Classes | Number of Pairs of Students |
|---|---|---|---|---|
| Section A | School A Classes 1,2 | 6 | School A[1] Classes 1,2 | 6 |
| | School B Classes 3,4 | 6 | School B[1] Classes 3 | 6 |
| | School C Class 5 | 5 | and 4 | 4 |
| Section B | School D Classes 6,7 | 6 | School D[1] Classes 5,6 | 6 |
| | School E Classes 8,9 | 7 | School E[1] Classes 7,8,9 | 7 |
| | School F Class 10 | 5 | School F[1] Class 10,11 | 5 |
| Not Tested | School G Classes 11,12 | | School G[1] Class 12 | |

The method of selecting pairs of CSMP students was as follows. At each school, the class lists of scores on the Kuhlman Anderson test (an ability test administered in January) were ordered from highest to lowest. Trios of students were then selected so that the members of the trio had almost identical scores and so that the trios were representative of the school from which they were selected. The number of trios selected was as shown above. From each trio, two students were tested and the third was an alternate. Then trios of Non-CSMP students were selected so as to match the CSMP trios.

For Section A the mean Kuhlmann Anderson raw scores were 74.2 for the 17 pairs of CSMP students and 75.6 for the 16 pairs of Non-CSMP students. For Section B, the scores were lower; 65.5 and 65.4 respectively.

Section A contained 2 parts and dealt with numerical processes, while Section B had 5 parts and was essentially non-numeric and related to classification and logical reasoning. The description of the test directions which are given in the next two sections (along with the actual results) are very brief. Actually the directions were quite long, with many demonstration materials and work sheets, and these directions are given in more detail in Appendices A and B.

The interviews were conducted by three people (two of whom were CSMP Evaluation staff members), and took place in rooms separate from the regular classrooms. They usually required about 45 minutes per student pair. Each interview was tape recorded, and these tape recordings (altogether about 50 hours worth), together with the interviewers' notes and the pages of student work, constituted the records of the interviews.

## Section A: Numerical Process

Part I

Each student was given a sheet of problems which had been "completed", i.e., the answer box had been filled in by "someone". Each student went quickly through his or her sheet and for each problem put either an x (for "pretty sure it's wrong"), or a ✓ (for "could be right"). Prior to doing this, two examples were done orally.

After both students had completed the page, which usually took 1-1½ minutes, each problem was reviewed. If an "x" had been recorded for a problem, that student was asked to explain why he/she was pretty sure it was wrong. If no response was then given, he/she was asked to estimate what should go in the box, or if still no response, whether the correct answer should have been larger or smaller. For many items, additional follow-up questions were prepared. Students could change their answers at any time.

After the first page was completed, a second page was done in a similar way.

For the analysis of the responses, each student pair was designated as being in one of three ability groups (high, medium or low) based on the Kuhlmann Anderson series. The table below shows the mean score on the Kuhlmann Anderson Test, and the mean time in seconds to complete each page of problems, for each ability group. It can be seen that the CSMP students were comparable to the Non-CSMP students in ability and in time required. Also the times required did not differ much for high, medium and low ability students. In fact "high", "medium" and "low" may be misleading. "High" corresponds to a percentile rank of over 90, but "low" means merely below average (percentile rank less than 50). On the average, the second page took about 10 seconds less than the first page.

Table 1

Mean Scores and Times
by Ability Groups

| | Number of Pairs | | Mean Raw Score Kuhlmann Anderson | | Mean Time for First Page | |
|---|---|---|---|---|---|---|
| | CSMP | Non-CSMP | CSMP | Non-CSMP | CSMP | Non-CSMP |
| High Ability | 3 | 3 | 94.2 | 93.8 | 84 | 69 |
| Medium Ability | 9 | 9 | 79.0 | 78.9 | 75 | 82 |
| Low Ability | 5 | 4 | 53.5 | 54.4 | 72 | 92 |
| Overall | 17 | 16 | 74.2 | 75.6 | 77 | 80 |

The next two pages show the problems of Part I. Beside each question are given the percentages correct for the high, medium and low groups and overall for CSMP (left hand column) and for Non-CSMP (right hand column). Where a follow up question was asked, it has been indicated by a phrase or two beside the question, together with percentanges correct across all CSMP and Non-CSMP students.

(For example, on the second problem, page 6 excerpted below, 76% of the CSMP students got the item right; 100% of the high ability group, 67% of the medium ability and 80% of the low ability group. When asked "four times <u>what</u> does equal 824". 95% of the CSMP students gave an answer in the 200's.)

| | | | |
|---|---|---|---|
| 4 x 106 = 824 | 100 | 80 | 4 x ☐ = 824: (95% and 50% of the students got an answer in the two hundreds) |
| | 67 | 83 | |
| | 80 | 100 | |
| | 76 | 88 | |

| | | |
|---|---|---|
| $26 + 36 = \boxed{512}$ | 100 100<br>100 83<br>60 75<br>88 84 | |
| $6 \times 13 = \boxed{53}$ | 67 00<br>39 50<br>70 75<br>53 47 | "What about 6 x 10?" Was this information useful for those who first thought given answer was correct: (75% and 100%) |
| $\boxed{47} - 14 = 33$ | 100 100<br>89 72<br>100 50<br>94 72 | |
| $58 + 62 = \boxed{120}$ | 100 100<br>89 67<br>80 75<br>88 75 | |
| $30 + 40 + 50 = \boxed{125}$ | 50 50<br>31 38<br>50 50<br>41 44 | Used the fact that the answer could not end in 5 (because 0 + 0 + 0): (33% and 44%) |
| $\boxed{192} + \boxed{192} + \boxed{192} + \boxed{192} = 428$ | 100 100<br>78 55<br>70 50<br>79 63 | |
| $\frac{1}{2}$ of $316 = \boxed{158}$ | 67 67<br>67 38<br>40 50<br>59 47 | Able to give procedure for testing whether 158 was correct: (75% and 67%) |
| $24 + \boxed{47} = 91$ | 83 100<br>61 72<br>50 62<br>62 75 | |
| $3,186 + 5,279 = \boxed{9,315}$ | 50 00<br>56 50<br>80 40<br>62 38 | |

$6,473 + 3,412 = \boxed{985}$

| | |
|---|---|
| 67 | 67 |
| 100 | 72 |
| 80 | 50 |
| 88 | 66 |

$1 \times 106 = \boxed{824}$

| | |
|---|---|
| 100 | 80 |
| 67 | 83 |
| 80 | 100 |
| 76 | 88 |

$4 \times \square = 824$: (95% and 50% of the student got an answer in the two hundreds)

$\boxed{30} - 43 = 13$

| | |
|---|---|
| 66 | 100 |
| 50 | 33 |
| 70 | 50 |
| 59 | 50 |

$\boxed{98} + \boxed{98} + \boxed{98} = 294$

| | |
|---|---|
| 50 | 50 |
| 56 | 50 |
| 20 | 75 |
| 44 | 56 |

Given that $98 + 98 + 98 = 294$ is true:
a) $97 + 97 + 97 = \square$ : (56%, 38%)
b) $\square + \square + \square = 297$: (50%, 28%)

$8 \times \boxed{12} = 160$

| | |
|---|---|
| 50 | 67 |
| 72 | 61 |
| 80 | 62 |
| 71 | 63 |

Given that $8 \times 12 \neq 160$,
Students were shown a 10 x 10 multiplication table: (This was useful for 80% and 60% of the students in finding out what 8 x 12 equals.)

$\frac{1}{2} \times 92 = \boxed{56}$

| | |
|---|---|
| 66 | 83 |
| 83 | 55 |
| 50 | 87 |
| 71 | 69 |

$\boxed{158} + \boxed{158} = 316$

| | |
|---|---|
| 100 | 100 |
| 78 | 66 |
| 50 | 62 |
| 74 | 72 |

Estimate what four 158's would be:
(56% and 42% gave an answer in the six hundreds)

$\boxed{600} + 3 + 13 + 7 + 8 = 731$

| | |
|---|---|
| 50 | 83 |
| 67 | 61 |
| 60 | 63 |
| 62 | 66 |

$53 + \boxed{20} = 33$

| | |
|---|---|
| 100 | 100 |
| 94 | 83 |
| 80 | 63 |
| 91 | 81 |

Idea of negative number: (38% and 0%)

The mean percent correct across all 18 problems is shown in Table 2, below. It can be seen that CSMP students in the medium ability range averaged 10 percent higher than similar Non-CSMP students, and that at the other two ranges of ability there was virtually no difference. Overall, there is a relatively small variation from high to low ability groups.

Table 2
Mean Percent Correct, Part I
By Ability Groups

| Ability Level | CSMP | Non-CSMP |
|---|---|---|
| High | 76 | 75 |
| Medium | 71 | 61 |
| Low | 65 | 63 |
| Overall | 70 | 64 |

There was very little pattern in which kinds of items CSMP or Non-CSMP students did better on. Altogether there were 9 items on which the overall percentage correct between the two groups differed by more than 10. These are shown below, by item type.

58 + 62 = 120 (88% correct for CSMP students, 75% for Non-CSMP)
24 + 47 = 91 (62% versus 75%)

192 + 192 + 192 + 192 = 428 (79% versus 63%)
98 + 98 + 98 = 294 (44% versus 56%)

$\frac{1}{2}$ x 316 = 158 (59% versus 47%)
4 x 106 = 824 (76% versus 88%)

47 - 14 = 33 (94% versus 72%)

6,473 + 3,412 = 985 (88% versus 66%)
3,186 + 5,279 = 9,315 (62% versus 38%)

It can be seen that CSMP students did much better with problems like the last two, but that otherwise it is difficult to generalize. It is worth noting that on three items (the last one above, 6 x 13 = 53, and 8 x 12 = 160) the percent correct, across CSMP and Non-CSMP combined, increases as one goes from high to low ability groups.

For those problems where the answer was indeed wrong, and where at least one of the pair of students correctly said so, the responses given to the question "Why are you pretty sure it's wrong?" were classified into one of three categories, described below in decreasing order of quality.

1. Adequate Reason or Estimate

   i) Adequate reason given
   ii) Reasonable numberical estimate given
   iii) Statement that correct answer should be larger (smaller)

2. Exact Mental Calculation

   Student attempted to calculate what the exact answer to the problem should be.

3. Incorrect or Insufficient Response

   i) Very general estimate of what correct answer should be (too general to be used in particular problem)
   ii) Wrong or inappropriate reason given
   iii) Response is irrelevant to problem
   iv) Unreasonable numerical estimate given

Obviously the categories are somewhat arbitrary and occasionally overlapped. Nevertheless it provided a useful way to summarize responses for various ability groups as is shown in Table 3 below. Where more than one response was given by a pair of students, only the best response was used.

Table 3
Mean Number of Responses
Per Pair in Three Main Categories

| | Adequate Reason or Estimate | | Exact Mental Calculation | | Incorrect or Insufficient Reason | | Combined | |
|---|---|---|---|---|---|---|---|---|
| | CSMP | Non-CSMP | CSMP | Non-CSMP | CSMP | Non-CSMP | CSMP | Non-CSMP |
| High Ability | 5.3 | 4.0 | 1.6 | 3.7 | 0.3 | 1.0 | 7.2 | 8.7 |
| Medium Ability | 4.1 | 4.3 | 1.4 | 3.0 | 0.7 | 2.3 | 6.2 | 9.6 |
| Low Ability | 1.6 | 3.8 | 2.8 | 1.3 | 1.2 | 2.8 | 5.6 | 7.9 |
| Overall | 3.6 | 4.1 | 1.9 | 2.6 | 0.8 | 2.1 | 6.3 | 8.8 |

It can be seen that at all ability levels, Non-CSMP students gave more responses than did CSMP students. If one translates the data in Table 3 to percentages of resposnes in each category, one gets the following:

| | Adequate Reason or Estimate | | Exact Mental Calculation | | Incorrect or Insufficient Reason | |
|---|---|---|---|---|---|---|
| | CSMP | Non-CSMP | CSMP | Non-CSMP | CSMP | Non-CSMP |
| High Ability | 74 | 46 | 22 | 43 | 04 | 12 |
| Medium Ability | 66 | 45 | 23 | 31 | 11 | 24 |
| Low Ability | 29 | 48 | 50 | 16 | 21 | 35 |

It can be seen that at the two higher ability levels CSMP students were more likely to give an adequate answer and less likely ot try to compute the exact answer than were Non-CSMP students, and this was reversed for low ability students. At all levels, CSMP students gave a lower percentage of incorrect/insufficient answers than did Non-CSMP students.

Part II

Part II of Section A was made up of 7 separate problems.

1. Estimating the Sum of 10 Numbers. Students were shown a card with the problem $5 + 5 + 4 + 6 + 3 + 3 + 6 + 6 + 5 + 4 = \square$ and the fact that there were 10 numbers was emphasized. They were then asked, in rapid succession, to estimate the largest and smallest the answer could be and, and to give their best guess for the answer.

   There was virtually no difference between the responses of CSMP and Non-CSMP students. Overall, about one-third of the "best guesses" were between 42 and 52 (the correct answer was 47), and this was also true for each ability range. About a quarter of responses were poor estimates (less than 22 or greater than 100) with proportionately more from the low ability students. It was also true that 72% of the low ability students (as opposed to only 29% of the other students) made a best guess" outside the largest-smallest bounds they themselves had given.

2. -1 Pattern. Students were shown a card with the problem: $6-5 + 9-8 + 2-1 + 5-4 = \square$. They were then asked to quickly write in the box their guess for the right answer. The percent of students who spotted the recurring -1 pattern, and hence got the answer to be 4, was nearly equal for CSMP and Non-CSMP students (41 to 44). Rather surprisingly the middle ability range of students was far more likely to get it (53%) than either the high (33%) or low (28%) groups, for both CSMP and Non-CSMP.

3. Reversing Digits. Students were shown the number 1,573 written on a piece of paper and asked to write the number obtained by reversing the 3 and 7. Then they were asked whether the new number was larger or smaller than the original and by about how much. Students were then asked the same questions for reversing the 3 and 5 of the original number, and then reversing the 3 and 1, except they were no longer allowed to write down the new number.

   Reversing the 3 and 5 proved most difficult, only 78% on whether the new number number was larger or smaller than the original number, while over 90% were correct on the other two reversals. Table 4 gives percentages of students who got the direction of change correct (was the new number smaller or larger?) and who got the amount of change correct (or about right; 30-50, 190-210, 1910-2010 respectively were the allowable ranges).

Table 4
Results from "Reversing Digits"

| | Mean Percent Correct: Direction of Change | | Mean Percent Correct: Amount of Change | |
|---|---|---|---|---|
| | CSMP | Non-CSMP | CSMP | Non-CSMP |
| High Ability | 89 | 94 | 78 | 89 |
| Medium Ability | 96 | 92 | 66 | 54 |
| Low Ability | 88 | 54 | 29 | 31 |

It can be seen that for low ability students, CSMP students did much better on the direction of change, but that both CSMP and Non-CSMP low ability students did very poorly compared to other students on the amount of change.

4. Grouping into Hundreds. The students were shown the following problem:

$1.22
1.81
1.51
1.53
1.32
1.33
1.39

They were told "Suppose we had to buy these 7 things and we want to be sure to take enough one dollar bills with us, but we don't want to take any more than we'll need". Then they were asked for a quick estimate, which they could then change after studying the problem. They were also asked to justify their answer.

The first answer each student gave was placed into one of three categories. The numbers of students in each category was as follows (the correct answer was 11)

| First Response | Number of CSMP Students | Number of Non-CSMP Students |
|---|---|---|
| 10,11,12 | 16 | 10 |
| 8,9,13,14 | 12 | 12 |
| Other | 4 | 7 |

Altogether in 5 of the CSMP pairs of students, and in 6 of the Non-CSMP pairs, one of the students described a strategy which involved grouping into hundreds (e.g., $ .22 + $ .81 is about a dollar).

5. 6 x □ = 138. The object was to take turns, using a calculator, in multiplying numbers by 6, until the one producing a product of 138 was found. The results were recorded each time and students encouraged to look at previous trials.

All but 2 of the CSMP pairs, and of the Non-CSMP pairs eventually got the correct answer without help. The mean number of trials required was 6.4 for both CSMP and Non-CSMP students. (Interestingly in 24 out of the 29 cases, the correct answer was suggested by the student who went first.) Six CSMP and 5 Non-CSMP students gave more than one bad guess. (A bad guess would be, say, 26, if 6 x 25 = 150 had already been established.)

After "6 x 23 = 138" was written, students were asked what they thought 6 x 24 would be (or 6 x 22, if 6 x 24 = 144 had been used previously). For both CSMP and Non-CSMP, 91% of the students realized the answer would be larger (smaller) than 138. About a quarter of each group got the correct answer.

6. ½ of 5,394 = □ . Students were asked how they could eventually solve this by using the calculator in a similar manner to #5 (i.e., the " ÷ " button was excluded). This turned out to be too hard for most students who were then asked to make a guess, and then to check the guess or see if they could find out whether the next guess should be larger or smaller.

Nine out of 15 (60%) of the CSMP pairs were able to suggest a reasonable verification process (x + x or 2x) versus 3 out of 15 (20%) of the Non-CSMP pairs.

7. Seven 69's. Students were shown the following "calendar".

| SUNDAY | MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY |
|---|---|---|---|---|---|---|
| 69¢ | 69¢ | 69¢ | 69¢ | 69¢ | 69¢ | 69¢ |

"It shows that everyday this week Bill is going to get 69¢ for doing chores". Students were then asked what would be the fastest way, on the calculator, to figure out how much Bill would earn by the end of the week.

69 x 7 or 7 x 69 was suggested by at least one of the students for 14 of the 16 (88%) CSMP pairs and for 8 of the 15 (53%) Non-CSMP pairs.

69 + 69 + ... was suggested by at least one of the students for 3 of the 16 (19%) CSMP pai s and for 12 of the 15 (80%) Non-CSMP pairs.

## Section B

### Part I - Twenty Questions Number Line

Students were to figure out what number, between 1 and 100, the interviewer had in mind by asking as few YES/NO questions as possible. The interviewer changed the number in mid-stream if necessary to avoid lucky wild guesses and discouraged random tries whenever possible. An acetate number line was marked to show progressively which numbers the student had eliminated. The students took turns and had 2 trials each.

For each trial several things were recorded. The means across the 18 CSMP and Non-CSMP pairs is shown below in Table 5. Each row says essentially the same thing: CSMP students did better on the first trial than Non-CSMP students, and this advantage disappeared on the second trial.

Table 5
Mean Scores on Part I

| | First Trial | | Second Trial | |
|---|---|---|---|---|
| | CSMP | Non-CSMP | CSMP | Non-CSMP |
| Number of guesses required to get number | 13.4 | 14.9 | 12.0 | 12.1 |
| Percent of numbers eliminated after 6 guesses | 62.7 | 50.2 | 64.0 | 63.4 |
| Numbers remaining when single-number quessing began | 33.7 | 44.4 | 23.4 | 22.2 |

The dominant strategy used in each trial was recorded and summarized in Table 6. Again it can be seen that while CSMP students more often used more efficient strategies (last two) on the first trial, there was no difference on the second trial.

Table 6
Number of Students Using Various Strategies

| | First Trial | | Second Trial | |
|---|---|---|---|---|
| Strategy Used | CSMP | Non-CSMP | CSMP | Non-CSMP |
| Single number guesses | 5 | 9 | 2 | 3 |
| Sng + intervals of 5,10 | 6 | 5 | 4 | 1 |
| Intervals of 5, 10 | 7 | 11 | 15 | 16 |
| Intervals of 10 + larger interval | 6 | 4 | 4 | 4 |
| Larger intervals | 12 | 7 | 13 | 12 |

Part II - Categorization of People Pieces (Figure 1, below)

The people pieces are a set of 16 4 x 6 inch cards on which are pasted pictures of children who are either boys or girls, tall or short, fat or thin, and red or blue. One people piece, for example, would be a tall, fat, blue boy. Some spare pieces (repeats) are also included. The students were told to "look at them and put (them) into...piles so that all of the cards in one pile are the same in some way and so that each pile is different from the others." Students took turns until neither could think of any other way to classify the cards.

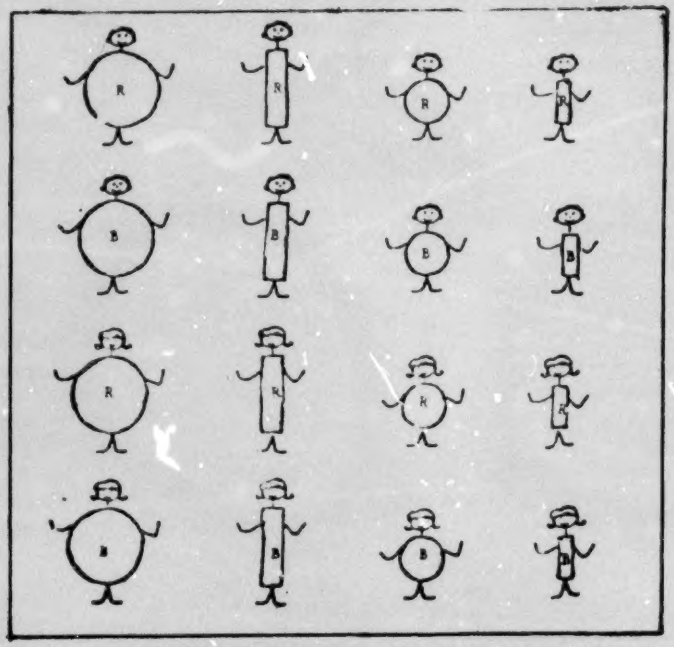


Figure 1. People Pieces; B = Blue, R = Red

The maximum "complexity of categorization" achieved by pairs of students is shown below. Complexity is defined as the number of dimensions which function in a given sort, and can range from 1 (for example, tall versus short) to 4 (all 4 dimensions function so there are 16 piles). The complexity was only counted as 4 dimensions when the student had progresses from 3 dimensions and laid out the pieces in some kind of pattern which showed up each dimension.

Table 7
Number of Pairs of Students
Achieving Various Levels of Complexity

| Complexity | CSMP | Non-CSMP |
|---|---|---|
| 1 dimension | 1 | 2 |
| 2 dimensions | 4 | 11 |
| 3 dimensions | 8 | 5 |
| 4 dimensions | 5 | 0 |

It can be seen that CSMP students as a group tended to make more complex sorts. The mean number of functioning attributes across all sorts was 1.7 for CSMP students and 1.4 for Non-CSMP. Ten of the 36 students spread or reviewed all the pieces before beginning their first sort compared to only 2 of the 36 Non-CSMP students.

Part III Concept Identification Task A (With People Pieces)

The interviewer had a "secret rule". There was a "YES" card for pieces which did fit this rule, and a "NO" card for pieces which did not fit the rule. The student handed the interviewer one piece at a time which was then placed in either the YES or NO column. Students were frequently asked if they knew what the rule was. Altogether there were four trials alternating between the two students. The rules, in order, were boys, blue girls, fat and tall, short.

Because students differ in the degree to which they must be sure of an answer before volunteering it, the following strategy was adopted to get a measure of how quickly they figured out the rule. The number of pieces required to figure out the rule was defined as: [total number of pieces required before student named the rule] minus [the number of successive correct (in the YES pile) pieces.] It is true that a student can check his hypothesis by trying a card that would not work but this happened rarely according to the interviewers. Table 8 shows that CSMP students required fewer pieces to solve the rule, particularly where the rule was 2-dimensional (Trials 2 and 3). It is interesting to note that Trial 2 was the easiest for CSMP students and the hardest for Non-CSMP students.

Table 8
Number of Trials Required to Solve the Rule

| | CSMP | Non-CSMP |
|---|---|---|
| Trial 1 - boys | 3.8 | 4.2 |
| Trial 2 - Blue girls | 3.1 | 5.9 |
| Trial 3 - fat and tall | 3.5 | 5.4 |
| Trial 4 - short | 4.4 | 4.2 |
| Total | 14.8 | 19.7 |

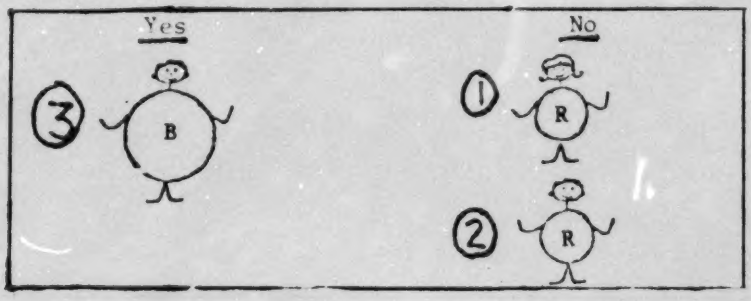
There was very little difference between CSMP and Non-CSMP students regarding the frequency with which: a) a card was repeated (in about 15% of the trials), b) a suggested rule contradicted established data (30%), and c) a student focused on one dimension instead of two, for example sex instead of sex and color, or vice versa (15%).

Part IV Concept Identification Task B

This was a standardized version of Part III. Instead of the student deciding what piece to offer for classification into YES/NO columns, he was shown the sequence of pieces that a "student at another school" used and which column (YES or NO) each went into. The sequences of pieces and the columns they went into is shown graphically below.

After the third piece (blue, tall fat boy) was placed, the student was asked to name as many rules as possible that could be right. (More "results" were shown to the student who then was again asked for possible rules. This process continued but unfortunately the administration protocol and scoring scheme devised for this question turned out to be not complex enough to allow an unambiguous interpretation later of what happened. Hence no data was analyzed beyond this one question after the third piece).

The mean number of suggestions per student pair for what the rule could be was 3.1 for both CSMP and Non-CSMP students. Of these, 0.5 (CSMP) and 0.4 (Non-CSMP) could not be correct because of the known data.



Part V Concept Identification Task C

This task used the people pieces augmented by a set of 8 green pieces. The YES card was removed. Again the students were to guess the rule. When a card was offered it was placed in the NO pile if it did not fit the rule, but was discarded face down if it did fit the rule. Thus the student could see a gradually increasing column of pieces which did not fit the rule. The first two cards in their NO pile were given by the interviewer. For the first trial, in which the rule was "girl", these given cards were the green tall thin boy and the red tall fat boy. For the second trial, in which the rule was "tall", the given NO cards were the blue short fat girl and the red short fat boy.

After the student had offered three pieces, he/she was asked to name the rule. The student then continued offering pieces and occasionally trying to name the rule. Thus the likelihood of the student volunteering a rule was influenced not only by his analytic abilities but also by his risk-taking tendencies.

The results can be summarized as follows:

Eventually got the rule: CSMP - 83%, Non-CSMP - 83%
Mean number of pieces required to get the rule: CSMP - 6.0, Non-CSMP 5.4

## Summary

It is always difficult to summarize data in which a wide range of responses is allowed, or where the question "why?" is frequently asked. Ideally one would like to participate in, or at least watch, the set of interviews. Then one could make up his or her mind as to how to evaluate the results, particularly insofar as they may reflect the differential effects of the CSMP curriculum.

Failing this possibility, the next best thing would be to watch videotapes, or at least read the transcripts, of all the interviews. This would run into hundreds of pages, but would give the reader/viewer an excellent overview about how third graders in general deal with various arithmetic processes; what kind of wrong answers they give, how fast they estimate and with what effect, what strategies are useful to them, etc. There are undoubtedly situations in which this activity would also provide the reader with a good picture of how CSMP and Non-CSMP students differ in these things. (Had the problems been related to the CSMP Minicomputer, no doubt the differences between CSMP and Non-CSMP would be apparent to the reader simply upon reading the transcripts.) However that is not true in the present case; the differences are not the kind that "hit you between the eyes". It has been necessary to summarize the responses item-by-item; gaining the capability to differentiate CSMP-Non-CSMP while at the same time losing the capability of reading a "story" about a single student pair from start to finish.

The results of this summarizing process have already been given and constitute the previous two chapters. The data are fairly objective, in the sense that one can fairly unambiguously classify the student responses as indicated.

1. For each of Section A and Section B, the results are based on interviewing 15-18 pairs of CSMP students and 15-18 pairs of Non-CSMP students, the pairs being very comparable in terms of ability test scores of student pairs, ability test scores of classes from which pairs are drawn, and location of schools containing these classes.

2. Section A

   a) In checking "completed" problems in which they were asked whether the given answer was wrong or "could be right" (Part I) CSMP students in the middle ability range (percentile rank 50-80) did 10 percentage points better on the average than Non-CSMP students. There was very little difference for high or low ability students.

   b) Non-CSMP students gave about 30% more responses when asked to explain why the given answer was wrong. Most of these additional answers were labelled "incorrect or insufficient reason" or "exact mental calculation" as opposed to the more desirable "adequate reason or estimate".

   c) On the series of seven problems in Part II, five of them produced either no CSMP-Non-CSMP differences:

      a) Estimating the sum of 10 numbers: $5 + 5 + 4 + 6 + 3 + 6 + 6 + 5 + 4 = \square$

      b) -1 Pattern: $6-5 + 9-8 + 2-1 + 5-4 = \square$

      c) Using hand calculator: $6 \times \square = 138$

      or very small ones in favor of CSMP students:

      d) Grouping into hundreds: $1.22$ / $1.39$ } 7 items

      e) Reversing digits: 1,573

d) On the other two problems there were fairly large differences in favor of CSMP student:

½ of 5,394 = ☐ : better able to check accuracy of first estimate

seven 69's: more likely to choose multiplication over additon as the fastest method of solution.

e) Mean scores were calculated for each student on various parts of Section A as shown in Table 9 below. The weighting scheme used to formulate these scores (not described here) was somewhat arbitrary, but probably reflects fairly well the students' performance.

Table 9

Mean Scores by Ability Levels, Section A

First Entry-CSMP Students, Second Entry=Non-CSMP Students

| | Number of Items | High Ability | | Medium Ability | | Low Ability | | Combined | |
|---|---|---|---|---|---|---|---|---|---|
| Part I ✓ or x[1] | 18 | 13.2 | 13.8 | 12.5 | 10.9 | 12.0 | 11.1 | 12.5 | 11.5 |
| Part I Reasons[2] | 13 | 4.3 | 3.0 | 3.1 | 1.1 | -0.8 | 0.8 | 2.4 | 1.4 |
| Part I Follow-up items | 8 | 6.0 | 7.0 | 5.9 | 4.4 | 3.8 | 2.8 | 5.4 | 4.5 |
| Part II | 21 | 15.7 | 14.8 | 15.7 | 12.4 | 12.4 | 9.3 | 14.7 | 12.1 |
| Total | 60 | 39.8 | 38.7 | 37.2 | 28.8 | 26.5 | 23.9 | 35.0 | 29.5 |

[1] For the given answer: probably alright(✓) or pretty sure it's wrong(x).

[2] (Number of adequate reasons) minus (number of insufficient/incorrect responses)

It can be seen that there was a difference in favor of CSMP for low ability students on Part II, virtually no differences for high ability students, and consistent differences in favor of CSMP for medium ability students. The combined standard deviations of the total scores was 6.8 for CSMP pairs and 8.3 for Non-CSMP pairs; thus the combined difference of 5.5 in favor of CSMP is close to one standard deviation. No statistical tests were computed because of the inappropirateness of using studnets, instead of classes, as the units of analysis.

a) In trying to determine a "secret number" between 1 and 100 with as few questions as possible, CSMP students used more efficient strategies in their first trial than did Non-CSMP students, but there was no difference on the second trial.

b) In sorting the 16 People Pieces into logically different piles, there was no difference in the number of sorts produced, though CSMP students tended to produce more complex sorts (involving more characteristics simultaneously).

c) In trying to determine the interviewer's "secret rule" by selecting People Pieces to be placed in a YES or a NO column, CSMP students required many fewer pieces, particularly when the rule used two dimensions of the pieces, than did Non-CSMP students.

d) In trying to determine the interviewer's secret rule by analyzing the People Pieces already given in YES and NO columns, there was no difference between CSMP and Non-CSMP students. This was also the case in Part V, when only the NO column was available for review and the presentation of cards was partially standardized.

e) Mean Scores were calculated for each part of Section B as shown in Table 10 below. Student pairs were divided into above average and below average according to the mean score on the Kuhlmann Anderson. The dividing line was approximately the 55th percentile; 7 pairs were above average and 11 below average in the CSMP group and also in the Non-CSMP group. Again the weighting scheme used to generate these scores was somewhat arbitrary.

Table 10

Mean Scores by Ability Levels, Section B

First Entry-CSMP Students, Second Entry-Non-CSMP Students

| | Number Of Items | Above Average | | Below Average | | Combined | |
|---|---|---|---|---|---|---|---|
| Part I | 10 | 7.4 | 6.8 | 5.7 | 4.6 | 6.3 | 5.4 |
| Part II | 2 | 1.2 | 1.0 | 0.6 | 0.5 | 1.1 | 0.6 |
| Part III | 8 | 4.3 | 4.6 | 5.5 | 3.6 | 5.1 | 3.9 |
| Part IV | 4 | 2.4 | 1.8 | 1.0 | 2.0 | 0.6 | 0.7 |
| Part V | 3 | 1.7 | 0.8 | 0.7 | 1.2 | 1.1 | 1.3 |
| Total | 27 | 17.0 | 14.9 | 14.2 | 11.8 | 15.3 | 13.0 |

It can be seen that CSMP students did better on the total score by an average of 2-3 points with similar results for above and below average students. The standard deviation of the total scores was 4.0 and 4.1 for CSMP and Non-CSMP students respectively, so that this difference was a little more than half a standard deviation. Again, significance tests were not carried out.

In summary then, there are very real differences in favor of CSMP students on a relatively small number of items. There is some evidence that above average CSMP students are better able to quickly check complete calculations; that CSMP students give fewer but better explanation for these calculations; that CSMP students are rather better at relating multiplication to division and addition; and that CSMP students are better able to "solve" a secret rule when required to choose the objects to be tested. For most of the other tasks however, the differences were not large enough to be of importance especially in view of the small numbers of students.

In the introduction it was stated that this study was exploratory. It is hoped that by viewing this process as more or less of a trial run, a more efficient procedure can be adopted in the future to further probe some of the more interesting findings.

## Appendix A

### Protocol for Administering Section A

Generally on the following pages a paragraph with a box around it indicates instructions to the tester. Otherwise the script is for the tester to say to the students. "Ⓘ" refers to the interviewer.

Part I - Arithmetic Processes
Materials - Problem pages 1 and 2
- Multiplication table
- 4 Demonstration Cards

Instructions to Students

I'm going to show you some problems that someone has done. I want you to look at each problem quickly and - without doing the whole calculation - tell me whether it could be right or whether you're pretty sure it's wrong. Let me show what I mean.

| Pass out demonstration card: | 3 x 9 = 512<br>1,626 + 2,497 = 4,123 |
|---|---|

S,I you read the first problem out loud. (Pause) Do you think it could be right or are you pretty sure it's wrong?

If student says 'wrong':

Yes, it's wrong. Even if we don't know the exact answer, we know that 512 is way to big.

If student says 'could be right':

Well how much would 3 tens be (Count by tens if necessary but get answer of 30.) So if 3 x 10 is 30, 3x9 couldn't be 512 - it's way too big.

So we mark that question wrong. How does your teacher mark an answer that's wrong. Well, let's do it the same way. You put an x (a check, circle, or whatever) beside the first problem.

S, II, you read me the next question (1,626 + 2,497 = 4,123)

Do you think it could be right, or are you pretty sure it's wrong.

If no quick answer, say:

Don't try to calculate the exact answer in your head. Just have a quick look and decide.

a) If either student says 'wrong', ask why they think it's wrong. The probable explanation will be that 1,000 + 2,000 ≠ 4,000, whence (I) should note that the 6 hundred and the 4 hundred would have carried so the answer would be 4 thousand and something. "We can't tell for sure whether or not it's exactly right, but it looks like it could be, so we say "could be right".

b) If both students say 'could be right':

That's right, we have 1,000 and 2,000 and we get one more thousand from carrying the 600 and 400. So that's 4 thousand and something altogether. We can't tell for sure whether or not it's exactly right but it could be, so we say 'could be right'.

How does your teacher mark an answer that is correct? OK, we'll do the same thing. You put a check (or whatever) beside this one to show it could be right - even though we don't know for sure.

Alright I want you to do the same for the problems on this page.

Hand out problem page 1 to each student.

Just look at each problem and quickly decide whether it could be right or whether you're pretty sure it's wrong. Don't try to calculate the whole answer. Let's see who finishes first.

If either student takes a long time with any question, say:

_____, you shouldn't take so long on a question. If you don't see anything wrong, just put a check (or whatever) for could be right, but don't calculate the whole answer.

After both students have completed problem page 1, go through the page item by item. Generally, we wish students to discuss the item with one another - argue if they have different answers, defend to (I) if they have the same answers. Where students have marked an item wrong they should explain their reasons. If they can't, they should give an estimate of what they think should have gone in the box. If they can't do this, they should at least indicate whether the correct answer should be larger or smaller than the given one.

On the next page problem page 1 is reproduced together with appropriate questions to follow up certain of the items.

$26 + 36 = \boxed{512}$ - If student responds "Could be Right", ask "what would 20 + 30 equal?"

$6 \times 13 = \boxed{53}$

$\boxed{47} - 14 = 33$

$58 + 62 = \boxed{120}$ - If student says "Pretty sure its's wrong" then after student estimate of correct answer "Suppose it was 50 + 60, then what would the answer be?" Get 110. Then, "Does that give you any ideas?"

$30 + 40 + 50 = \boxed{125}$ - If X, but no good reason, or if calculated exactly, "What about this one." (Show Demonstration Card 1, page 27)

$\boxed{192} + \boxed{192} + \boxed{192} + \boxed{192} = 428$ - If CBR, "If this (point) was just 28, what would go in the box?"
- If X, but no explanation "What would this (point to 428) have to be for 192 to be right? Take a guess."

$\frac{1}{2}$ of $316 = \boxed{158}$ - If CBR, "How could we check exactly?"

$24 + \boxed{47} = 91$ - If CBR, "What is 20 + 40?" Get 60. "Does that give you any ideas?"

$3,186 + 5,279 = \boxed{9,315}$ - If CBR, "But doesn't 3,000 + 5,000 = 8,000 So shouldn't our answer be 8,000?

NOTE: - SINGLE UNDERLINED ITEMS SHOULD BE READ BY STUDENTS, ALTERNATELY, EACH TIME
- DOUBLE UNDERLINED ITEMS SHOULD BE READ WITH RESPONSE OF "COULD BE RIGHT"
- OTHER ITEMS READ BY STUDENTS AT DISCRETION OF INTERVIEWER

Hand out problem page 2 to each student.

Just look at each problem and quickly decide whether their answer could be right or whether you're pretty sure it's wrong. Don't try to calculate the whole answer. Let's see who finishes first.

"Take up" problems in same way as problem page 1

On the next page, problem page 2 is reproduced together with appropriate follow-up questions.

$6{,}473 + 3{,}412 = \boxed{985}$

$4 \times 106 = \boxed{824}$ — If X, with good explanation, "It would have to be 4 times what, do you think to equal 824?"

$\boxed{30} - 43 = 13$ — If CBR, "Are these two the same" (show Demonstration Card 3, page 25)

$\boxed{98} + \boxed{98} + \boxed{98} = 294$ — If X, "No that's correct". Then go to a).
If CBR, "That's right" and go to a).
a) "If 98 + 98 + 98 = 284 (point) "What would 97 + 97 + 97 equal?"
b) "What number would have to go in each of these boxes (show Demonstration Card 2, page 27)

$8 \times \boxed{12} = 160$ — Regardless of answer, but after getting students who said "wrong" to estimate the correct answer, "Let's see if we can use this multiplication table to figure out what 8 x 12 is?"

$\frac{1}{2} \times 92 = \boxed{56}$ — If X, and reasonable explanation, "What would you guess this (point to 92) would have to be to make 56 the right answer?".

$\boxed{158} + \boxed{158} = 316$ — If CBR, "That's right. Now what if I were adding 4 of these 158's instead of 2 of them. What would the answer be?"

$\boxed{600} + 3 + 13 + 7 + 8 = 731$ — If X, "How would we figure out what should actually go in the box?" (What is wanted is process of adding together 3 + 13 + 7 + 8 and subtracting that sum from 731.)

$53 + \boxed{20} = 33$ — If CBR, "But 53 + 20 = 73, so 20 doesn't work. What number <u>should</u> go in the box?"

If X, "Do you see anything funny about this one?"

Interview A, Part I

Demonstration Cards for Part I:

1.

$30 + 50 + 80 + 70 + 60 = \boxed{325}$

2.

$\boxed{98} + \boxed{98} + \boxed{98} = 294$

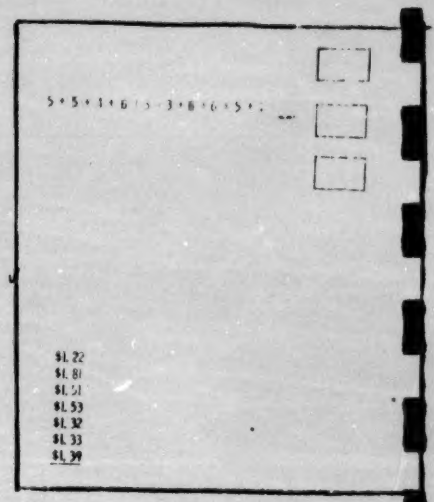
$\square + \square + \square = 297$

3.

$30 - \boxed{43} = 13$

$43 - \boxed{30} = 13$

## Student Interview - Section A



Part II - Estimation
Materials - 2 sets of problem pages
- 1 hand calculator

Instructions:

Here are some problems the student forgot to do.

Hand out problem page

1. In this first problem there are 10 numbers to add together. And off to the side there are 3 boxes. In the first box you both put what you think is the largest the answer could be. Don't try to do the whole problem in your head. Just put down what you think is the largest the answer could be.

   Now in the second box you both put what you think is the smallest the answer could be. The smallest the answer could be.

   Now in the third box you put down your guess for what the answer is.

After both students have completed all three boxes, say to the student with the best "largest" estimate (smallest number ≥ 50):

_____, you put _____ in the first box. That was a good guess for the largest the answer could be. Would you tell us why you chose that answer?

If no sensible explanation is given, ask the other student the same question. Or can try, "why can't it be, say, (pause) 60".

Now before I tell you the answer, would either of you like to change your guess for what the answer actually is - in the third box. Just put your new answer below the boxes.

If any changes are made, note them and ask why they changed.

OK, the answer is 47, so your guesses were a little too high (or pretty good, or a little too low, etc.)

When students are completing boxes encourage them to put down a quick answer and not to mentally compute exact answer. On the other hand, allow lots of time for them to explain their answers or to change their guess in the last question. If they do change their guess without explanation, ask if they did it in their head (as a way of provoking an explanation).

2. Now turn the page over and look at the problem at the top. Put down your guess for the right answer - write it in the box.

If there is more than 3 or 4 seconds' pause:

You're not supposed to calculate it all out. Just put your guess in the box.

a)

If either student put an answer less than 10:

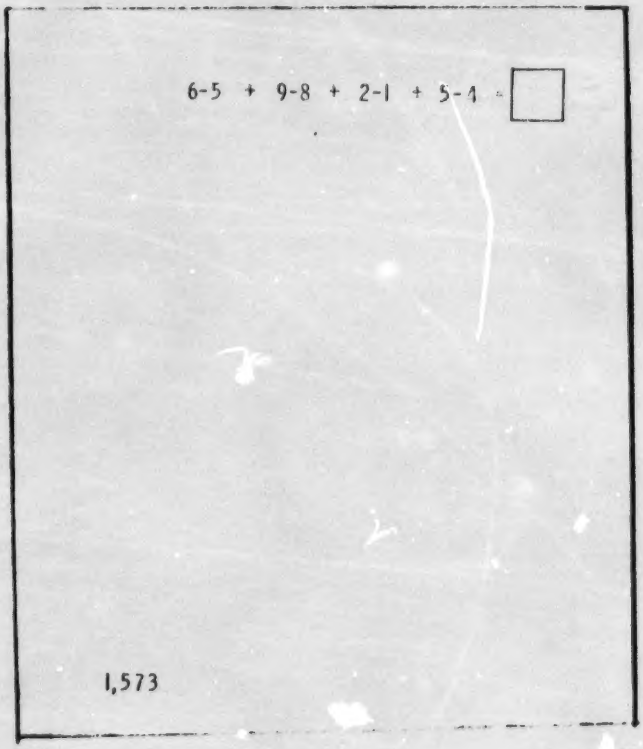
_____, that was a very good answer. Tell us why you chose that answer.

b)

If neither student put answer < 10 OR if no explanation involving successive differences of 1 is given:

Alright now listen carefully while I read the problem to you in a special way: Six take away five, (pause), plus, nine take away eight, (pause), plus, etc. Now, who thinks he knows a better answer.

Give both students some time to think about it. Then call on one who wants to answer (but note it, if both students want to answer), and get his new answer and then his explanation. If neither student sees the pattern, briefly tell them and go on to #3.



3. Now look at the next number that's shown;

| Point to "1,573" on each student page. |
|---|

Let's make a new number, which will be almost the same as this one, except we'll switch the 7 and the 3. You write down what the new number would be. (Pause and check). Now, is our new number bigger or smaller than the one we started with? (Pause) _____, what do you think?

| If "bigger", ask other student: |
|---|

| If "smaller", ask: | How much smaller would you say? Why__smaller? And _____, what do you think? |
|---|---|

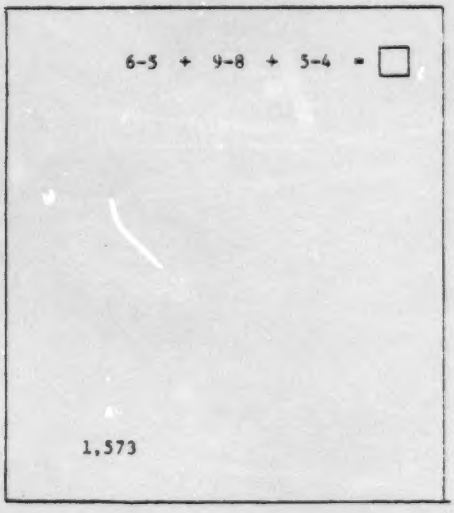
Cross out the number you wrote. Now let's start with this number (point to "1,573") again, and this time we won't write down our new number - we'll just think about it. This time we'll switch the 5 and the 3. Will the new number be bigger or smaller than the one we started with?

| Repeat above procedure except "bigger" is the answer to be followed up. If a response of "about 200" or "200" is given, say: |
|---|

Well, that's a very good answer. But it's not exactly 200 smaller. Would you say it's more likely to be 202 smaller or 198 smaller?

This time, let's switch the 1 and the 3 in our starting number. Now what do you think?

| Repeat above procedure. If a student says about 2,000 bigger, tell him/her that it is a very good answer. Can they give an even better answer? |
|---|



Interview A - Part II

4. Turn the page over and look at the dollars and cents shown at the bottom of the page. Suppose we had to buy these 7 things and we want to be sure to take enough one dollar bills with us - but we don't want to take any more than we'll need. How many one dollar bills would we need? (Pause 2 seconds).

S II, how many would we need to take?
and S ?, how many do you think we'll need?

Now, both of you study the question and see if you want to change your answer. But don't try to do the whole problem. I won't give you that much time.

The likely sequence of answers is:
$7 or $8 - (looking only at dollar amounts)
$9 or $10 - (looking at dollars and adding $1 for 20+80 and $1 for 50+50)
$10 or $11 - (as above and adding one for 32+33+39)

After both students have given second answer, ask student with best answer to explain how he got it. Then ask other student if he agrees, or can suggest an even better answer. Try to distinguish the above 3 cases, plus whether or not student is allowing a dollar for the left over cents (11¢ actually).

Interview A - Part II

5.

If necessary, rearrange seating, so that students are sitting together, facing interviewer, for remaining questions.

Now I have a problem that we'll try with this calculator.

Show problem page with 6 x ☐ = 138

Look at where it says six times box equals 138. Let's take turns trying different numbers to put in the box. _____, you give me a number to try and I'll multiply it by six on the calculator.

Okay, _____. Six (press '6') times (press 'x') _____(press _____) equals (press '=') _____. So our answer is _____. Let's keep track of our answers.

When using calculator, do it upside down and slowly, so students can see what is being done. Can be speeded up as you proceed. When writing down answers, do it in a column, on your sheet of paper - turning upside down for students to see each time after writing it. As you write it, say: '_____, says 6 times ___ equals ____.' Alternate students. After a while your page might look like this:

6 x ☐ = 138

6 x 12 = 72
6 x 30 = 180
6 x 20 = 120

If both students give two bad answers in a row - i.e., outside previously established bounds - help the students by suggesting 20 and do 6 x 20 = 120. Encourage students to look at the answers they got with earlier trials.

After correct result has been achieved, ask other student what he thinks the answer to 6 x 24 would be - saying it aloud as you write it under 6 x 23 = 138. (If 6 x 24 has already come up during iteration, use 6 x 22). Then ask the same question of other student.

6. Now look at this problem.

| Turn problem page over and point to $\frac{1}{2}$ of 5,394 = ☐ |
|---|

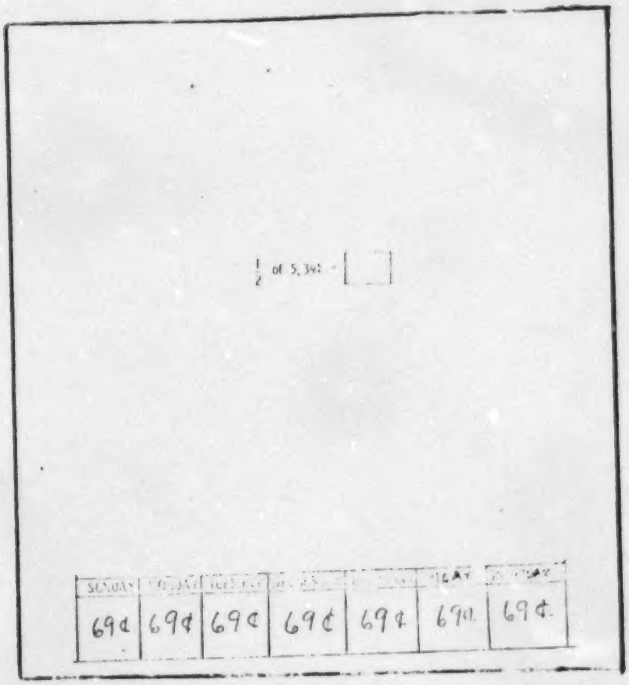
Think about this for a minute and see if you can think of a way we could use the calculator to figure it out - like we did the last one. This is a very hard one. (Pause) Would you like to take a guess?

a) If no good response:

Well let's leave that one - it's very hard.

If student 'tests' an answer, say x, by 2x or x+x, do it on the calculator write down the calculation below the question and ask him what he would try next. Then ask other student what he would try next. Don't do either one on calculator but just encourage students by saying:

That's very good. We don't have time to finish this problem, but I know you two would have been able to get it if we did have time.



J. Okay, here is the last problem:

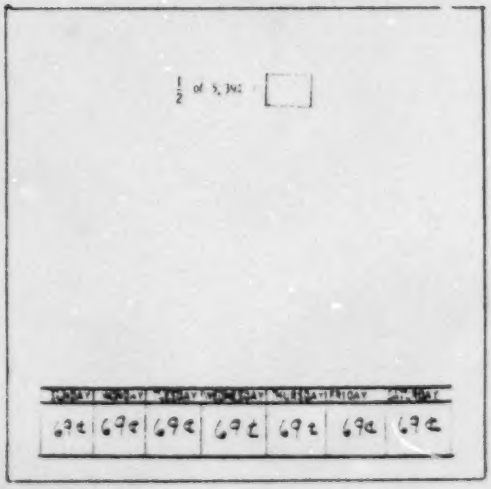
Show bottom half of page.

This is a calendar. It shows that every day this week Bill is going to get 69¢ for doing chores. How much will he have by the end of the week (point to the Saturday entry)? Now how could we use the calculator to figure out the answer? (Pause) What would be the fastest way we could do it on the calculator?

Take a response from either student. If 69+69+. . . . is suggested say:

Yes, we could get the answer that way. But I think there's an even faster way to do it. See if you can figure out what it is.

If 7x69 or 69x7 is given, praise to the sky, do it on calculator for them and terminate interview. If not suggested, tell them to watch carefully, and, saying as you do it, press 7x69=483 - ("That was a hard one, wasn't it', and terminate interview.)



Appendix B

Protocol for Administering Section B

Generally on the following pages a paragraph with a box around it indicates instructions to the tester. Otherwise the script is for the tester to say to the students.

Part I - "Twenty Questions" - Number Line

Time: 15 minutes

Materials: Acetate number line from 1 to 100

Instructions to Students:

"I'm going to think of a number from 1 to 100 but I'm not going to tell you what it is."

Show number line from 1-100.

- "I want you to figure out which number on this number line I'm thinking of by asking me questions I can answer 'yes' or 'no'."
- "Try to ask as few questions as you can."
- "We'll do this one at a time, starting with (name of student)."
- "After you figure out what number I'm thinking of I'll think of a new number and (name of other student) will try to figure out my number."
- "He/she will try to do it by asking fewer questions than you."
- "We'll keep track of how many questions you ask by writing them on this paper, and I'll check 'yes' or 'no' depending upon my answer."
- "You can cross-out the number you've already asked me so that you won't ask me the same question twice."

"O.K. - I picked my number - you can ask me any question that I can answer with a 'yes' or 'no'.

(I) may have to change his number depending upon the student's luck in random guessing. This is done so that the student is not reinforced for wild guesses. (I) should also choose the larger of two domains so that the student does not prematuraly narrow the range (e.g., if student's first question is "Is it between 1 and 25, (I) should answer "no"). If the student is eliminating ten numbers each time, progressively moving up (or down) the number line, (I) should answer "yes" to the second-to-last decile.

If student asks: "Is it big", ask him (her) to show you what she/he means by way of the number line.

Following five continuous one-number questions, (I) should say: "It will take a long time to figure out my number this way. Remember - you can ask me <u>any</u> question I can answer 'yes' or 'no'.

If student narrows range to 15 or fewer numbers, and then begins to <u>systematically</u> eliminate numbers one at a time, accept the third one. If student narrows range to 8 or few numbers and then begins to <u>randomly</u> eliminate numbers one at a time, accept the third one. Otherwise after the <u>second</u> random try, say: "No, it will take a long time to figure out my number this way".

When first student has determined (I)'s number, say

"O.K. - that took you _____ questions. Let's see if (other student) can do it in fewer questions."

Two trials for each student (A,B,A,B

4~

**Part II** - Categorization of People Pieces

**Time:** 5 minutes

**Materials:** Twenty 4 x 6 inch cards on which are pasted "people pieces" which vary along the dimensions of height (tall and short), color (red and blue), shape (fat and skinny) and sex (male and female).

Instructions to Students:

"I have a lot of cards here with different figures on them, see? I want you to look at them and put these cards into categories - or piles - so that all of the cards in one pile are the same in some way, and so that each pile is different from the others. You can make as many different piles as you'd like".

If neither of the students understands the directions, (I) should sort the cards according to color and explain what he has done.

"You can take turns. (Student's name), would you like to start?"

When both students, taking turns, can't think of any new categories take TALL FAT RED GIRL, show it to one of the students, saying:

"Describe this person for me. What can you tell about this person?"

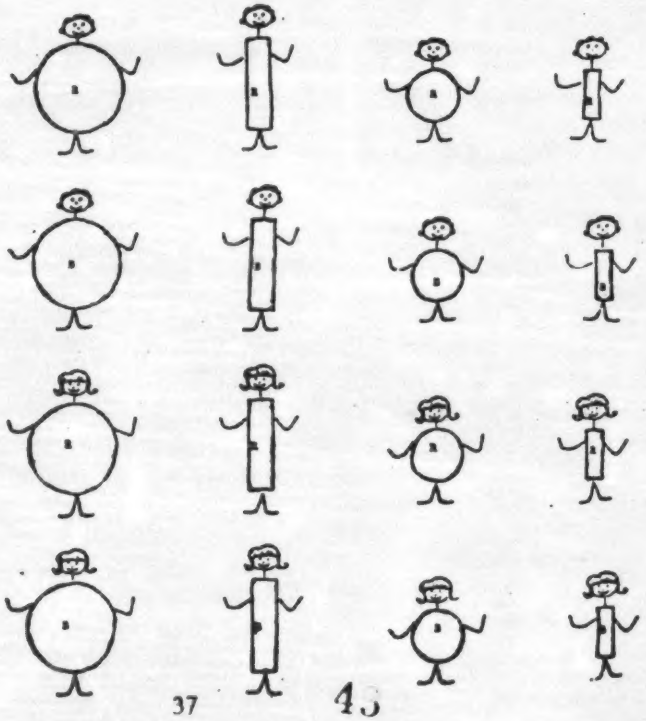
Assist if necessary, or get other student to help, but make sure the correct descriptor for each dimension is said.

Repeat with SHORT FAT RED GIRL, then with TALL THIN BLUE GIRL

If student starts categorizing according to long hair - short hair; circles - rectangles; etc., point out that these are the same as the girl-boy; fat-skinny categories.

(I) should record by hand the students' categories, but not repeat them back to student.

People Pieces



Part III - Concept Identification Task - A
Time: 10 minutes
Materials: People pieces plus two 4 x 6 inch cards; one with the word "yes" printed on it, the other with the word "no" printed on it.

Instruction to Students:

- "Let's do something different. This time, I'm going to think of a secret rule, and you have to figure out what my rule is."
- "You won't know what my rule is at first, but there's a way to figure it out after awhile."
- "I want you to hand me any card, one at a time, to try to figure out my rule."
- "I will let you know if it fits my rule by putting it under the 'yes' sign if it does and under the 'no' sign if it doesn't."
- "You'll see how it works while we're playing the game."
- "You want to hand me as few cards as you can before you guess my rule."
- "Let me know when you think you've figured out my rule."

"Alright, let's start with (student's name) and you'll see how it works."

Whenever appropriate, and at least by the fourth card, ask:

"What do you think my secret rule could be?" ... "Let's keep going and find out."

If student thinks he/she has exhausted all cards in secret rule but has not (all short and fat when category is fat, for example, say:

"Keep going. There are still some more that fit my rule".

For non-chance taking students who appear to have rule, but won't verbalize it, ask what he/she thinks rule is as soon as you suspect he/she has figured it out. If student gives correct-rule, stop; if no response or incorrect response) continue. It will be necessary, for students' sake, to give a "score" - just be fair. Main criteria for later scoring is to determine at what point student had rule figured out - so ask frequently.

Part IV - Concept Identification Task - B
Time: 3-4 minutes
Materials: Same as Part III (People Pieces shown below plus YES/NO cards)

Instructions to Students:

"This time I'm going to show you which cards a student at another school picked. I want to see if you can figure-out what my rule was when I played the game with him. You'll both be playing the game this time, so I want you to both watch carefully".

1. "First, he showed me this card (Red Short Fat Girl), and I put it in my 'no' pile."
2. "Then he gave me this card (Red Short Fat Boy) and I put it in my 'no' pile."
3. "Then he handed me this one (Blue Tall Fat Boy) and I put it in my 'yes' pile."

   "What do you think my rule could be?"

   > Take responses from both students. Rule is blue, but at this point, "tall" or "tall and blue" are also correct.

4. "Let's go on. Next, he gave me this one (Red Tall Thin Girl) and I said 'no'."
5. "Now where do you think I put this one? (Blue Tall Thin Boy)

   > Take responses from both students

6. "How about this one?" ( Blue Tall Fat Girl )

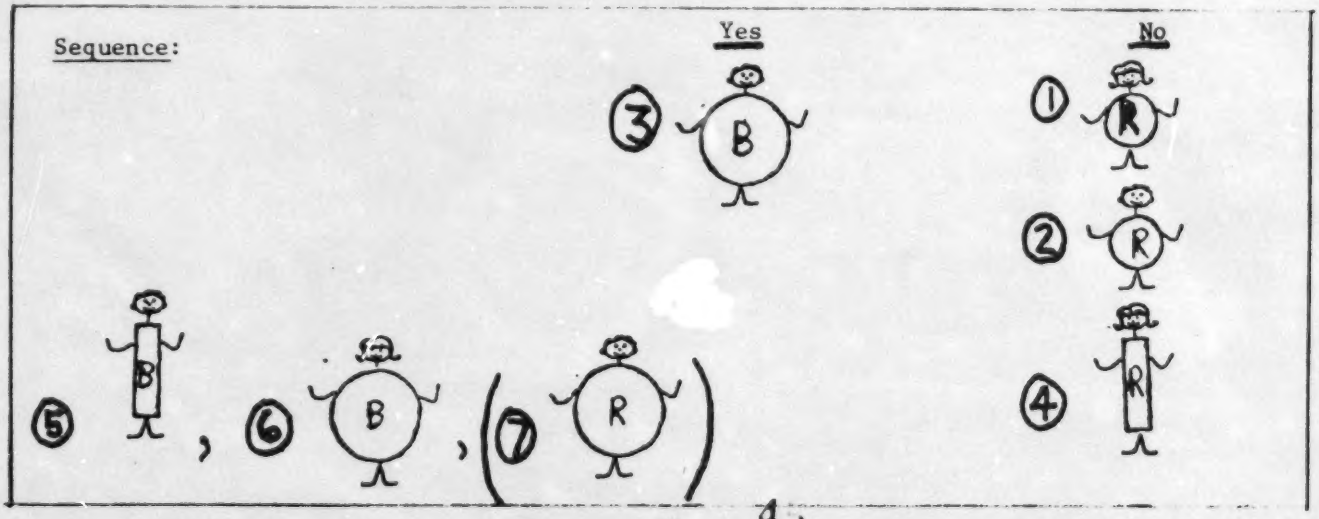
   "What do you think my rule could be?"

   > If both agree on the correct rule, praise students and end game. If not, go on .

7. "Where do you think I put this one (Red Tall Fat Boy)?"
8. "Now what do you think my rule is?"

   > Take responses and end game.

Sequence:



Part V - Concept Identification Task - C

Time: 5 minutes

Materials: People Pieces, plus a set of green people pieces, plus the "No" card.

Instruction to Students:

Conspicuously add green cards.

"This time we'll have green people too and I want you to figure out what my rule is by seeing what my rule is not.

Conspicuously remove the "YES" card.

"That's why we'll only have a 'no' pile".

I. "Let's start with (student's name).

This card (Green Tall Thin Boy) does not fit my rule.

Neither does this card (Red Tall Fat Boy).

Now you hand me one card at a time to try to figure out what my rule is."

Rule=Girl

If student hands (I) card which does not fit his rule, (I) should place it in the 'NO' pile.

If it does fit his rule (I) should say 'Yes, this fits my rule' and turn it face down.

If student asks to see it, remind him that s/he is only allowed to see the ones that don't fit your rule.

After student places 3 cards, ask:

"What do you think my rule could be?" ..."What else?"

If correct: "That's right. Now it's (other student's turn)." (2nd Trial)

If not completely correct or not possible to know: "Well, let's keep going and see."

If it becomes apparent that student is looking for cards that fit into your "NO' pile, caution him against this, stating:

"Remember, you want to find out what my rule is - not what fits in my 'NO' pile.

II. Second Trial. Rule=Tall

First card: Blue Short Fat Girl

Second Card: Red Short Fat Boy

Proceed as above.

46